INDIVIDUAL ANALYSIS: CORRUGATED FIN DESIGN

For the Red Feather Capstone Project Fall 2020

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Introduction

The Red Feather Capstone team has been tasked with the creation of solar air heater or solar furnace. This device uses solar radiation absorbed by metal fins to generate heat. Air is forced past the fins in an enclosed box to convect heat from the fins and then into a building. The previous Red Feather Capstone team found in their analysis that a fin system, figure 1.d, generated the most heat out of the four designs they tested.

Figure 1. Previous Team's Prototypes. From left to right: (a), (b), (c), (d) [1]

The current team would like to see if they can further improve on the fin design to generate even more heat. One method observed in many commercial options is to increase the surface area of the fins with corrugations. To do this the increased surface area will be determined and then the absorbed solar energy compared. The fabrication cost for type will then be compared to the difference in energy generation to determine if the changes are worth the extra cost of production.

Assumptions

Some assumptions will be made to simplify the analysis of the system. First the solar irradiance and impact angle will be assumed to be steady and constant. This ignores the effects of clouds and the change in angle due to the motion of the sun. The team plans to place the fins in the furnace at an angle of 35˚ which will place the fins and furnace perpendicular to the sun's rays for the most time throughout the year [2]. However, as the angle of the sun drops throughout the day and the max angle changes through the year the fin's effectiveness will be reduced, with the fins casting shadows on each other. This calculation will be avoided since it can typically be assumed that the increase in Surface area will make up for the shadows and still perform better than a flat plate [2]. The furnace could also be tilted throughout the year to make up for the change in angle of the sun making further justifying this assumption.

The second assumption will be to ignore any affect the acrylic panel or box will have on the light passing through it due to warping or shadows as both plate designs would experience the same effect inside the furnace.

Finally, the corrugated metal will be assumed to be a repeating pattern of perfect triangles. The corrugations will have more rounded edges where bent, changing the overall surface area very slightly, but ultimately a difference between corrugations versus none is still demonstrated well.

Calculations

Surface Area Comparison

First the increased surface area of the corrugated fins will be calculated. The type metal brake available to Red Feather can create angles up to about 120˚ however, the steeper the angle the more difficult it is to create a pattern. 60˚ was found to be easily produced into a pattern the length needed (3ft) without catching on the break. This also creates an equilateral triangle with easy dimensions to calculate for scaling purposes. The 60˚ bend creates equal side lengths, l where the height is simply.

$$h = l * \frac{\sqrt{3}}{2} \tag{1}$$

The height dimension is important for fitting the fins in the box as they must be able to be able to be angled properly and fit between the back and the top plus any insulation or other features. A basic design of the corrugation pattern is shown in figure 2.

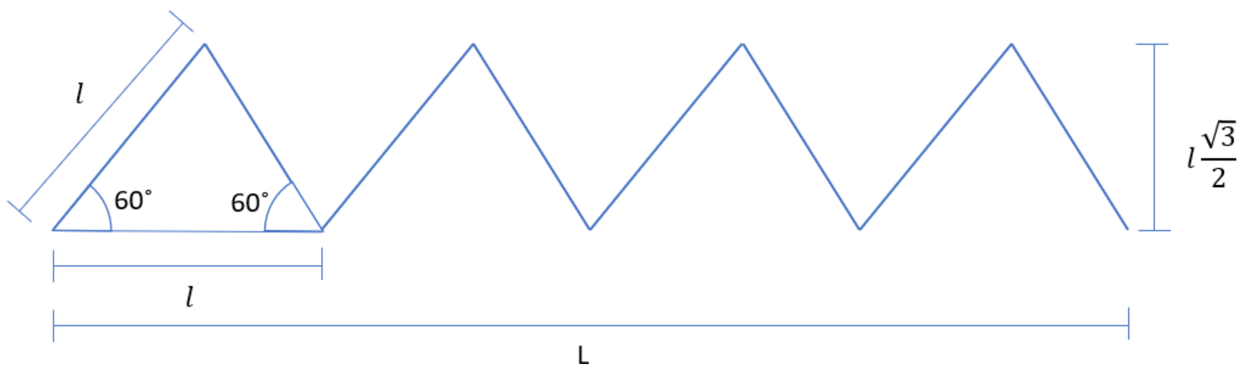

Figure 2. Corrugation Pattern

With the pattern known, the increase in surface area can also be easily calculated. As the base of the triangle is the same as the side lengths, each corrugation simply doubles the surface area per patterned unit. So, for each fin the total surface area would be:

$$SA = 2 * L * W \tag{2}$$

Where L is the length measured from one end of the fin to the other along a straight line and W is the width of the fin.

Energy Comparison

For slow moving air the energy removed by convection due to air flow over a surface can be estimated by:

$$q''_{conv} = 0.22(T_s - T_\infty)^{4/3} \text{ [3]} \tag{3}$$

Where: T_s is the temperature of the surface

T_∞ is the temperature of the ambient air

For a flat plate of metal exposed to the sun and this convection an energy balance can be expressed as:

$$q" = \alpha_s G_s + \varepsilon\sigma T_{sky}^4 - 0.22(T_s - T_\infty)^{4/3} - \varepsilon\sigma T_s^4 \text{ [3]} \quad (4)$$

Where: α_s is the solar absorptivity of the surface

ε is the emissivity of the surface

T_{sky} is the atmospheric temperature

σ is the Stefan Boltzmann constant

G_s is the solar irradiance

Solar absorptivity, α_s, is a measure of a surface's ability to absorb heat energy from the solar radiation. It is directly related to how reflective a surface is compared to a black body, a theoretical completely unreflective surface for which $\alpha_s = 1$ [3]. Table 1 shows several types of surfaces and their solar absorptivity values with black-painted metal surfaces having a value of 0.97. This is the color of choice for the Red Feather team.

Table 1. Solar Absorptivity and Emissivity of Common Surfaces [3]

Surface	α_s	ε (300 K)
Evaporated aluminum film	0.09	0.03
Fused quartz on aluminum film	0.19	0.81
White paint on metallic substrate	0.21	0.96
Black paint on metallic substrate	0.97	0.97
Stainless steel, as received, dull	0.50	0.21
Red brick	0.63	0.93
Human skin (Caucasian)	0.62	0.97
Snow	0.28	0.97
Corn leaf	0.76	0.97

Emissivity, ε, is a measure of the ability of a surface to emit heat compared to a blackbody where $\varepsilon = 1$ [3]. It also depends on the surface color and reflectivity but is also depends slightly on material type. Per table 1, for a black-painted metal surface, the value is also 0.97 meaning the surface can radiate heat as well as it can absorb it.

For an ambient temperature, T_∞,of 30˚C, the atmospheric temperature, T_{sky}, can be assumed to be about -10˚C [3]. The temperature of the surface, T_s, will be assumed to be 100 ˚C as the furnace has yet to be tested, but this is a common temperature for a black surface on a sunny day [3].

The Stefan Boltzmann constant, σ, relates the temperature of a surface to the emissivity and is equal to $5.67\ X\ 10^{-8}\ W/m^2 \cdot K^4$.

Solar irradiance, G_s, is the measure of solar energy available per unit surface area emitted by the sun. The idealized average is 1366 W/m^2 [4].

With these values the energy absorbed by the flat surface can be computed using equation 4:

$$q" = (0.97)(1366\ W/m^2) + (0.97)\ (\sigma)(263\ K)^4 - 0.22(100 - 30)^{4/3} - (0.97)\sigma(373\ K)^4 = 460 W/m^2$$

It can then be assumed that per the surface area increase demonstrated in equation 2 that for the corrugated design equation 4 would become:

$$q" = \alpha_s 2G_s + \varepsilon\sigma T_{sky}^4 - 0.22(2)(T_s - T_\infty)^{4/3} - \varepsilon\sigma T_s^4 \quad (5)$$

With equation 5, the energy can be calculated for the corrugated plate:

$$q" = (0.97)2(1366\ W/m^2) + (0.97)\,(\sigma)(263\ K)^4\ - 0.22\,(2)(100-30)^{4/3} - (0.97)\sigma(373\ K)^4$$

$$= 1721\ W/m^2$$

Thus, it is shown that the corrugations dramatically increase the effectiveness by doubling the surface area for a plate with the same linear dimensions as the flat plate as the energy per unit area increases faster than the losses per unit area.

Fabrication Cost Comparison

From the analysis the corrugations are about 3.74 times more effective at producing heat energy than a flat plate when completely exposed to sunlight. The corrugations, however, require double the material of the flat fins which will double the cost per fin and take far longer to fabricate. Assuming the flat fins would require 5 cuts in each of the two 3'x3' sheet metal stock used to make 12, 3'x6" fins and each cut takes about 2 minutes to make it would take 20 minutes to fabricate the fins. To make the corrugated fins, four 3'x3' sheets will be bent 36 times (every 2 inches) each to make the corrugations taking an estimated 10 minutes per sheet and then cut taking 40 minutes for a total estimated fabrication time of 80 minutes.

The aluminum sheets cost $22 each [5] and assuming a hourly rate for a worker of $20 per hour the flat fins cost can be calculated as:

$$2(22\ \$) + \left(20\frac{\$}{hr}\right)\left(\frac{1}{3}hr\right) = \$50.67$$

The fabrication cost for corrugated fins can be calculated as:

$$4(22\ \$) + \left(20\frac{\$}{hr}\right)\left(\frac{4}{3}hr\right) = \$114.67$$

Which shows that the corrugated fins are about 2.26 times as expensive to fabricate compared to the flat fins.

Results/Recommendations

With the corrugated fins costing 2.26 times more but being 3.74 times more effective, it is recommended that the team should pursue the corrugated design. After the results of this analysis were presented to the team members, they agreed to pursue the corrugated fins for the increased effectiveness. Corrugated designs are used in many commercial options, including those previously purchased by Red Feather, which helps justify the analysis and decision to pursue them as well.

References

[1] "Red Feather Solar Furnace 2," Northern Arizona University, Jan-2020.

[2] "Best Angle for Solar panels in the US - Calculations and ZIP Codes", Axion Power, 2020. [Online]. Available: https://www.axionpower.com/knowledge/solar-panel-angle/#:~:text=Ultimately%2C%20What%E2%80%99s%20the%20Best%20Angle%20For%20Solar%20Panels,the%20country%20is%20between%2025%20and%2035%20degrees. [Accessed: 09-Oct- 2020].

[3] T. Bergman, A. Lavine, D. DeWitt, F. Incropera and T. Bergman, Incropera's principles of heat and mass transfer, 8th ed. John Wiley & Sons, Inc, 2017, pp. 753-764.

[4] "NASA - SOLAR IRRADIANCE", Nasa.gov, 2020. [Online]. Available: https://www.nasa.gov/mission_pages/sdo/science/Solar%20Irradiance.html. [Accessed: 09- Oct-2020].

[5] "36 in. x 36 in. Plain Aluminum Sheet in Silver", homedepot.com, 2020. [Online]. Available: https://www.homedepot.com/p/M-D-Building-Products-36-in-x-36-in-Plain-Aluminum-Sheet-in-Silver-57000/100351161. [Accessed: 09- Oct- 2020].